to include a broader range of frequencies as the Reynolds number of the jet was increased. In fact, at the highest Reynolds number, spectra were obtained such as shown in Fig. 4 with a broad frequency content including numerous discrete frequency peaks. It appears that over the Reynolds number range of these experiments there remained a good deal of orderly structure in the fluctuations even where simple amplitude measurements would indicate a turbulent flow situation. In view of these results, and those of Crow and Champagne,[4] it is reasonable to expect some orderly structure in the oscillations of supersonic jets of much higher Reynolds numbers.

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Series Expansion of the Eccentricity for Near Parabolic Orbits

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Introduction

THE determination of the elements of a near parabolic orbit is difficult because of the slow convergence of the series involved. This Note provides a convergent series to compute the eccentricity of a near parabolic orbit in terms of the pericenter distance q and the time of flight t through an angular distance of $\pi/2$ rad from the pericenter (Fig. 1). For a planet flyby, these quantities can be accurately determined by onboard radar measurements and observations. In the figure, the elliptic, parabolic, and hyperbolic orbit, all having the same pericenter, are denoted by E, P, and H, respectively. The elements of the elliptic, or hyperbolic, orbit are to be computed, while the parabolic orbit is introduced as a reference orbit. The key equation for the expansions is a hypergeometric equation.

The time of flight from the pericenter, along an elliptic orbit, is given by

$$(\mu/a^3)^{1/2}\, t = E - e\sin E \qquad (1)$$

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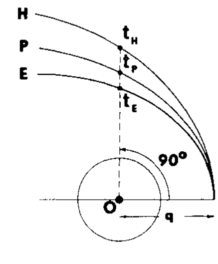

Fig. 1 Nomenclature.

and for a hyperbolic orbit

$$(\mu/a^3)^{1/2}\, t = e\sinh H - H \qquad (2)$$

while for a parabolic orbit, we have

$$(\mu/2q^3)^{1/2}\, t = \tan(v/2) + \tfrac{1}{3}\tan^3(v/2) \qquad (3)$$

where μ is the gravitational constant, a is the semi-major axis, v is the true anomaly, E is the eccentric anomaly, and H the hyperbolic anomaly. Since

$$\cos v = (\cos E - e)/(1 - e\cos E),$$
$$\cos v = (e - \cosh H)/(e\cosh H - 1) \qquad (4)$$

when $v = \pi/2$, we have

$$\begin{aligned} e &= \cos E, \quad \sin^2(E/2) = (1-e)/2 \\ e &= \cosh H, \quad \sinh^2(H/2) = (e-1)/2 \end{aligned} \qquad (5)$$

Then, the time of flight from the pericenter to one end of the semi-latus rectum, along an elliptic orbit, is

$$t_E = (2q^3/\mu)^{1/2}(E - \sin E\cos E)/4\sin^3(E/2) \qquad (6)$$

and for a hyperbolic orbit

$$t_H = (2q^3/\mu)^{1/2}(\sinh H\cosh H - H)/4\sinh^3(H/2) \qquad (7)$$

On the other hand, if the flight is along the parabolic orbit, we have

$$t_P = \tfrac{4}{3}(2q^3/\mu)^{1/2} \qquad (8)$$

Assume that the pericenter distance q, and the time of flight t to one end of the semi-latus rectum along either an elliptic or hyperbolic orbit are known. Hence t_P can also be computed from Eq. (8).

Let

$$\phi = \tfrac{5}{2}(\mu/2q^3)^{1/2}(t_P - t) \qquad (9)$$

Then, from Eqs. (6–8)

$$\phi = \tfrac{10}{3} - 5(E - \sin E\cos E)/8\sin^3(E/2) \qquad (10)$$

for the elliptic case, or

$$\phi = \tfrac{10}{3} - 5(\sinh H\cosh H - H)/8\sinh^3(H/2) \qquad (11)$$

for the hyperbolic case.

Since ϕ is known, the transcendental equation [(10) or (11)] could be solved for E or H and subsequently the eccentricity of the orbit could be obtained from Eq. (5). For near parabolic orbit, E and H are small and a direct computation is sensitive to error. We propose to find a convergent series for the computation of E and H and subsequently, of the eccentricity e, in terms of ϕ. For this purpose, let

$$\begin{aligned} \theta &= \sin^2(E/2), \quad \text{elliptic case} \\ \theta &= -\sinh^2(H/2), \quad \text{hyperbolic case} \end{aligned} \qquad (12)$$

By taking the derivative of ϕ with respect to E, we have

$$\tan(E/2)\, d\phi/dE + \tfrac{3}{2}\phi = 5 - 5\cos(E/2)$$

Using Eq. (12) to change the independent variable from E to θ

$$\theta(d\phi/d\theta) + \tfrac{3}{2}\phi = 5 - 5\cos(E/2) \qquad (13)$$

By taking the derivative of this equation with respect to θ, we have

$$\theta(1-\theta)\, d^2\phi/d\theta^2 + \tfrac{5}{2}(1-\theta)\, d\phi/d\theta = \tfrac{5}{2}\cos(E/2) \qquad (14)$$

By eliminating $\cos(E/2)$ between the two equations, we have the hypergeometric equation[1]

$$\theta(1-\theta)\, d^2\phi/d\theta^2 + (\tfrac{5}{2} - 2\theta)\, d\phi/d\theta + \tfrac{3}{4}\phi = \tfrac{5}{2} \qquad (15)$$

A similar process in the hyperbolic case results in the identical equation.

This equation is satisfied by the hypergeometric series, absolutely convergent for $|\theta| < 1$

$$F(a,b,c,\theta) = 1 + \frac{a\cdot b}{1\cdot c}\theta + \frac{a(a+1)b(b+1)}{1\cdot 2\cdot c(c+1)}\theta^2 + \frac{a(a+1)(a+2)b(b+1)(b+2)}{1\cdot 2\cdot 3\cdot c(c+1)(c+2)}\theta^3 + \ldots \qquad (16)$$

where

$$a = \tfrac{3}{2}, \quad b = -\tfrac{1}{2}, \quad c = \tfrac{5}{2} \qquad (17)$$

From the definition (10) or (11) of ϕ, it is seen that when E or $H = 0$, $\theta = 0$, $\phi = 0$, and $d\phi/d\theta = 1$. Hence the solution of Eq. (15) satisfying these conditions is

$$\phi = \tfrac{10}{3} - \tfrac{10}{3}F(\tfrac{3}{2}, -\tfrac{1}{2}, \tfrac{5}{2}, \theta) \qquad (18)$$

Explicitly, we have

$$\phi = \theta + \frac{5}{28}\theta^2 + \frac{5}{72}\theta^3 + \frac{25}{704}\theta^4 + \ldots + \frac{5\cdot 1\cdot 3\cdot 5\ldots(2n-3)}{n(2n+3)\cdot 2\cdot 4\cdot 6\ldots(2n-2)}\theta^n + \ldots \qquad (19)$$

The recurrence formula for computing the coefficient a_n of θ^n of this series is

$$a_n = \frac{(2n+1)(2n-3)}{2n(2n+3)}a_{n-1} \qquad (20)$$

Reversing the series

$$\theta = \phi - \frac{5}{28}\phi^2 - \frac{5}{882}\phi^3 - \frac{1075}{543312}\phi^4 - \ldots \qquad (21)$$

Since $\theta = (1-e)/2$ in both the elliptic and the hyperbolic case, we have the final series for the eccentricity valid for either the elliptic or the hyperbolic orbit.

$$e = 1 - 2\phi + \frac{5}{14}\phi^2 + \frac{5}{441}\phi^3 + \frac{1075}{271656}\phi^4 + \ldots \qquad (22)$$

By the convergent property of the hypergeometric function, it is seen that the series is absolutely convergent when $|\theta| < 1$, that is, for

$$e < 3 \qquad (23)$$

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Vibrations of Sandwich Plates under Uniaxial Compression

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Introduction

Vibrations of sandwich plates have been investigated by many authors.[1] Most of the papers that appear in the literature restrict the analysis either to symmetric sandwich plates (having identical faces), or to plates taking the membrane energy of the faces alone. Among earlier investigators, Hoff[2] first included the bending rigidity of the faces for studying the bending and buckling behavior of symmetric sandwich plates. Recently, Rao and Nakra[3] considered the effects of the bending rigidity of the faces as well as rotary, longitudinal translatory, and transverse inertias in their analysis for the case of free vibrations of sandwich plates with dissimilar faces. However, the influence of edge forces on the vibrations of a sandwich plate has not been studied in detail. To the authors' knowledge, only Shahin[4] presented a few results on the free vibrations of sandwich plates in the presence of mostly in-plane tensile loads treating the faces as membranes. The results are inadequate, as they fail to identify the behavior of the plate, particularly when the edge loads are compressive in nature.

A detailed study of the dynamic behavior of unsymmetric sandwich plates (having dissimilar faces) under uniaxial compression, taking the bending energy of the faces into account, has been presented in this Note. The effect of variation of various nondimensional parameters has also been investigated.

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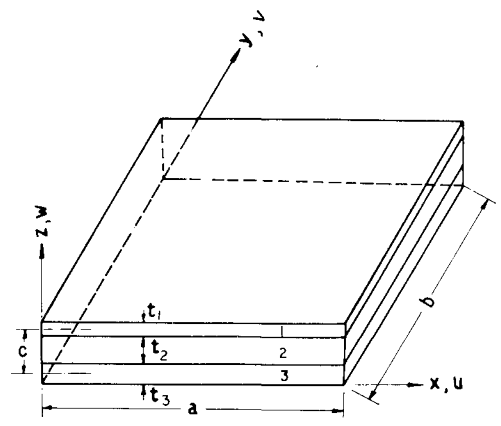

Fig. 1 Geometry of sandwich plate.

Governing Equations and Solution

As an extension of Ref. 3, one can obtain the equations of motion for the vibrations of an unsymmetrical sandwich plate (Fig. 1) under uniaxial compression accounting for the inertia term only. These are presented below:

$$S_1\left(u_1'' + \frac{1+\nu_1}{2}v_1'^* + \frac{1-\nu_1}{2}u_1^{**}\right) + \frac{S_2}{t_2^2}(cw' - u_1 + u_3) = 0 \qquad (1)$$

$$S_1\left(v_1^{**} + \frac{1+\nu_1}{2}u_1'^* + \frac{1-\nu_1}{2}v_1''\right) + \frac{S_2}{t_2^2}(cw^* - v_1 + v_3) = 0 \qquad (2)$$

$$S_3\left(u_3'' + \frac{1+\nu_3}{2}v_3'^* + \frac{1-\nu_3}{2}u_3^{**}\right) - \frac{S_2}{t_2^2}(cw' - u_1 + u_3) = 0 \qquad (3)$$

$$S_3\left(v_3^{**} + \frac{1+\nu_3}{2}u_3'^* + \frac{1-\nu_3}{2}v_3''\right) - \frac{S_2}{t_2^2}(cw^* - v_1 + v_3) = 0 \qquad (4)$$

$$(D_1 + D_3)\nabla^4 w - S_2\frac{c}{t_2^2}\{c(w'' + w^{**}) - (u_1' + u_3' - v_1^* + v_3^*)\} + \rho\ddot{w} - N_x w'' = 0 \qquad (5)$$

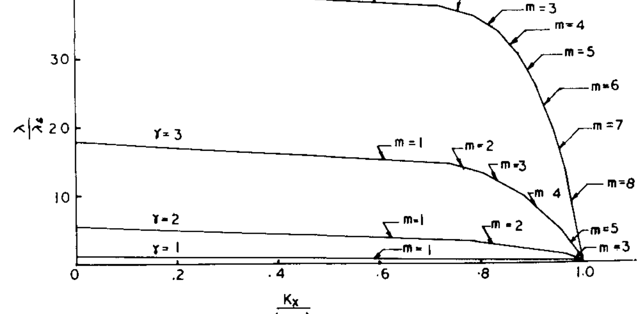

Fig. 2 Variation of frequency ratio with axial load ratio.